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International Journal of Solids and Structures 42 (2005) 1669–1680
INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES
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Dynamic behaviour of a belfry caused by the swinging bells

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Received 30 January 2004; received in revised form 16 August 2004
Available online 6 October 2004

Abstract

Through non-destructive tests we have determined the dynamic characteristics of the wall belfry of Our Lady of the Carmen Church (Valencia, Spain). We explain the method applied to obtain these characteristics, knowing the spectrum response of the structure when one of the bells turns. After studying the bells, we have determined the fundamental magnitudes of their oscillatory movement: weight of bells, turn speed, unbalance and inertia. These values are necessary to determine the variable time forces that they introduce in the structure. Using a numerical model at real scale of this structure we have determined the least favourable dynamic amplification factor (DAF) and the resistant response of this tower in a quasi-static analysis for these forces.

Keywords: Numerical simulation; Parameter identification; Model approximation; Monitoring; Vibration measurement; Structural stability; System response

1. Introduction

Knowing the time-dependent forces induced by the movement of the bells is of major relevance in the design, project or restoration of bell towers (Schutz, 1994). Depending on their angular velocity and their unbalance, these forces may have important dynamic interactions with the supporting structure.

The three more extended systems for sounding bells are the Central European, the English and the Spanish, shown in Figs. 1–3 respectively. In the first of them, the bells tilt on their axis, while in the English system they complete full circles, alternating the sense of rotation at each cycle. Both systems are highly

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0020-7683/$ - see front matter
doi:10.1016/j.ijsolstr.2004.08.014

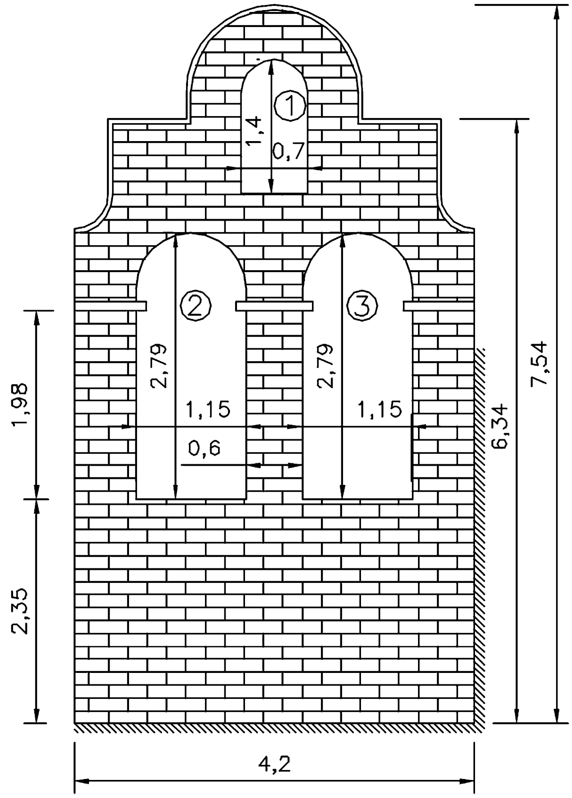

Fig. 1. Wall belfry elevation. Bell distribution.

unbalanced and the bells are located inside a tower with structures specifically designed to that effect. Conversely, in the Spanish system, a counterweight provides a high level of balance (see Fig. 3) and the bells, directly anchored on the tower windows, rotate continuously in the same direction.

This study aims to determine the dynamic effect produced when the bells turn on the structures of the wall belfry of Our Lady of the Carmen Church in Valencia, Spain. This Church belongs to the Carmelite Order. Its construction began in 1841 and it was finally inaugurated in December 11, 1891.

The church consists of a plant of three naves and a transept. On one of the lateral naves a wall belfry is constructed with three openings, each one with its corresponding bell. This wall belfry reaches a height of 7.54 m on the mentioned lateral nave. See Figs. 1 and 2.

Contrary to the usual disposition, the wall belfry, not placed on the main facade, has its base standing on top of one of the first two buttresses as seen from the front. The side wall of the main nave provides it with a lateral support as a 3.5 m overlap in height exists between that wall and the belfry. Furthermore, its base which is factually embedded in the roof of the lateral nave supplies stiffness on the direction of the axes of the naves.

The wall belfry has three openings to house bells. The two biggest have a width of 1.15 m and a height of 2.79 m; the third one has a width of 0.7 m and a height of 1.4 m. It has a width in the base of 4.2 m and a constant thickness of 1.0 m.

The biggest bell, "Sta. Teresa of Jesus" is located in the lower opening next to the main nave, in the area in which the wall belfry joins to the lateral wall of this nave. The lightest bell: "Felipe" is housed in the

Fig. 2. Wall belfry photography.

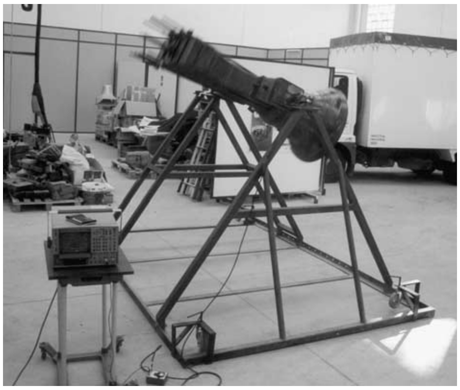

Fig. 3. Laboratory tests. ''Sta. Teresa'' bell.

highest opening. The distribution of the bells in the wall belfry is the best for the correct transmission of the stresses introduced by them on the wall belfry.

On this wall belfry the bells movement consists of the complete rotation of the bell around the own axis. Impulse motors simulate manual control. The bells do not rotate at constant speed. A small force is introduced even in the axis at the moment the bell is with its ''mouth'' up, later on, gravity pulls them down. This movement can be simulated by a compound pendulum with big oscillations around its axis:

$$I \cdot \ddot{\varphi}(t) + m \cdot g \cdot d \cdot \sin \varphi(t) = 0 \qquad (1)$$

where I is the moment of polar inertia of the bell referred to the turn axis, m is the mass, g is the gravity, d is the centre of gravity position, and φ is the widespread coordinate that represents the rotated angle.

The solution of Eq. (1) is

$$\varphi(t) = 2 \cdot \arcsin\left[\beta \cdot \mathrm{sn}\left(\sqrt{\frac{g \cdot m \cdot h}{I}} \cdot t\right)\right] \tag{2}$$

where sn is the elliptic Jacobian function, β is a constant depending on the initial conditions given by the expression:

$$\beta = \sqrt{\frac{\left(\omega_0^2 + \frac{2 \cdot g \cdot m \cdot d}{I} \cdot (1 - \cos\omega_0)\right) \cdot I}{4 \cdot g \cdot m \cdot d}} \tag{3}$$

where the initial free fall speed is the initial speed indicated in Table 1, for each bell:

$$\omega_0 = \frac{n \cdot \pi}{t} \quad (\mathrm{rad/s}) \tag{4}$$

n is the number of clapper blows, t is the time (in s).

As consequence of this oscillatory movement, each bell introduces time variable forces—horizontal and vertical—on its supports. Eq. (5) represents approximately the horizontal forces:

$$H(t) = m \cdot d \cdot [(\dot{\varphi})^2 \cdot \sin\varphi(t) - \ddot{\varphi} \cdot \cos\varphi(t)] \tag{5}$$

The characteristic values for each bell have been obtained through laboratory tests (Heyman and Threfall, 1976; Ivorra and Llop, 2002). See Fig. 3. They are described in Table 1.

The simulated horizontal force obtained from Eq. (5) is represented in Fig. 4.

A frequency analysis of the variable forces induced by the swinging of bells is needed to assess the reliability of the supporting structure. Due to their geometry, materials used in their construction

Table 1
Characteristic of the bells

	1	2	3
Unbalance* (m)	0.028	0.071	0.115
Swing velocity (rad/s)	6.59	4.5	3.45
Max horizontal force (N)	71.5	353	1016
Max vertical force (N)	467	1703	4798
Weight (N)	382.6	1265	3502

Distance between centre of rotation and centre of gravity.

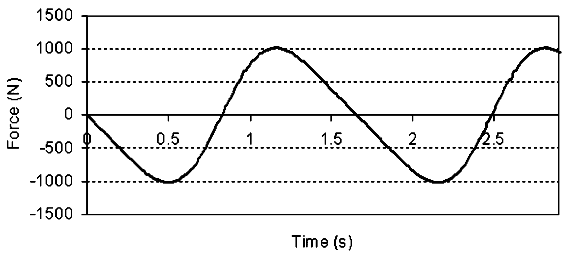

Fig. 4. Horizontal force introduced by the biggest bell on its supports.

and the traditional disposition of the bells, most towers show significantly higher stiffness to axial loading than to torsion or bending, making therefore more critical the horizontal component of the induced forces (Wilson and Selby, 1993; Bachmann et al., 1995). The Fourier analysis of each one of the functions that represent the simulated horizontal force introduced by the three bells requires to evaluate the possible dynamic amplification factor. Through this study it is possible to analytically evaluate the harmonics of each bell.

2. Description of the dynamic test

The test was carried out while the restored bells were fitted with new wooden yokes and impulsive motors. To measure the frequency response of the dynamically excited structure, an accelerometer was applied at the base of the opening of the "Felipe" bell, in such a way that we registered perpendicular accelerations to the wall belfry. The three bells introduce horizontal movements on this structure in the same direction due to their arrangement on the belfry.

The accelerometer was positioned in the middle of the axis of the wall belfry with the purpose of determining the influence of the bending, trying to eliminate it by recording the possible influence of vibrations due to torsion.

Values were registered only when the hardest bell—"Sta. Teresa of Jesús"—turns, since this is the one that "a priori" induces the highest stresses on the wall belfry.

The value of the first characteristic frequency of the system can be approximately calculated from the expression for a system with a single degree of freedom without damping. This value allows to fix the range of characteristic frequencies of the system

$$f_N = \frac{1}{2 \cdot \pi} \cdot \sqrt{\frac{K}{M}}$$

$$K = \frac{3 \cdot E \cdot I}{L^3}$$

where f_N is the natural frequency, K is the system rigidity, M is the mass, I is the geometric moment of inertia, and L is the wall belfry height.

The spectrum presented in Fig. 5, allows to deduce several clearly identifiable values: 0.6625 Hz, 1.1375 Hz; these values approach to the values proposed for the first and the second modes of the "Sta. Teresa of Jesus" bell. A small discrepancy exists among these values and those proposed in Table 2 (0.59 Hz, 1.18 Hz), since they have been calculated theoretically. Once identified these two values of the spectrum response, the next frequency will be necessarily corresponding to the first harmonic of the belfry. That is the characteristic frequency of the system flexion: 1.3125 Hz.

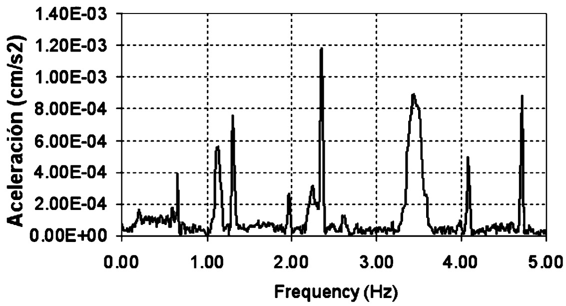

Fig. 5. Spectrum response of the wall belfry.

Table 2
Harmonic corresponding to the horizontal forces of each bell

Name	1° Harmonic (Hz)	2° Harmonic (Hz)
(1) Felipe	1.195	2.39
(2) Sta. María Gracia Plena	0.787	1.57
(3) Sta. Teresa of Jesús	0.59	1.18

The value of the damping ratio of the system has been determined through Eqs. (6) and (7) by means of the evaluation of the decreasing widths, obtaining $\varepsilon = 0.015$

$$\delta = \ln \frac{X_N}{X_{N+1}} \tag{6}$$

$$\xi = \frac{\delta}{\sqrt{4 \cdot \pi^2 + \delta^2}} \tag{7}$$

where δ is the damping logarithmic decrement and ξ is the damping ratio.

3. Interaction wall belfry-bells

With the purpose of evaluating the stresses induced by the bells, a three-dimensional finite elements model to real scale was used; the elastic characteristics of the pattern will be adjusted through the results obtained in the dynamic test. The commercial computer program SAP2000™ has been used with three-dimensional finite elements of eight nodes: SOLID (Wilson, 2000). This model possesses 1868 nodes and 1068 SOLIDS, with an allowed total number of degrees of freedom of 5388. The application of this non-destructive dynamic test will allow to adjust the value of the mean elasticity module of the structure

$$[m] \cdot \{\ddot{x}\} + [k] \cdot \{x\} = \{0\} \tag{8}$$

The system of Eq. (8) has a solution different from the trivial one when:

$$E = \omega_i^2 \cdot k_i \cdot \rho \tag{9}$$

where E = material elasticity modulus; ω_i = natural frequency; k_i = constant, which depends on the mass, the rigidity and the discrete model; and ρ = average specific weight of the material.

Since the structure is built with solid brick masonry, it is considered to have a bulk density of $1.77 \times 10^8\,\mathrm{N/m^3}$, as it proposes the Spanish norm NBE AE-88 (1999). After an iterative process of adjustment the average elasticity of this wall belfry, obtained from the displacements originated by deflexion in its weak plane, is the following (Table 3).

Once the numeric model was adjusted (Fig. 6), the effect of the different bells on the structure were studied, analysing when these bells turn separate and when they turn together.

Table 3
Characteristic frequencies on the numeric model and mean elasticity

Vibration mode	Adjusted frequency (Hz)	E (N/mm^2)
1	1.312	1250
2	2.097	”
3	4.000	”
4	6.791	”

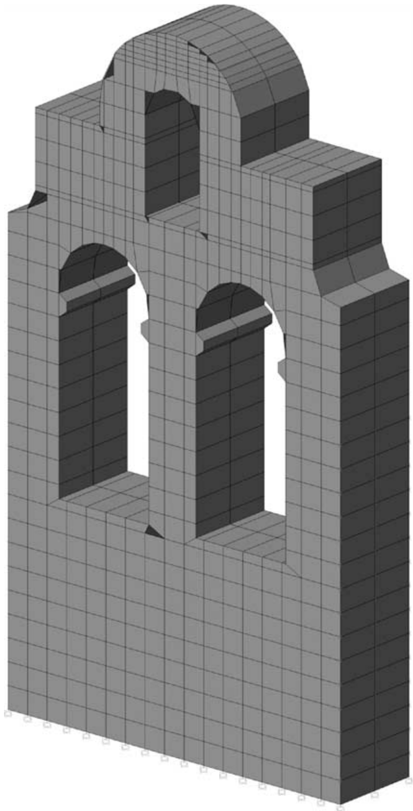

Fig. 6. Numerical model of the wall belfry.

In a first phase the response of the structure was analysed considering the action of the "Sta. Teresa" bell, with the purpose of evaluating the dynamic amplification factor introduced by the bell swinging. In Table 4 the results are presented.

In this instance, the system undergoes a forced vibration as it is described in (10)

$$[m] \cdot \{\ddot{x}\} + [k] \cdot \{x\} = \{f(t)\} \tag{10}$$

These results allow to affirm that the total horizontal force introduced by the "Sta. Teresa" bell for a quasi-static study is 0.39 times its weight, 1366 N

$$F = \mathrm{DAF} \cdot \frac{H_{\max}}{P} \tag{11}$$

Table 4
Displacements when the "Sta. Teresa" bell turns

Point	Max static (mm)	Max dynamic (mm)	Dynamic amplification factor
Coronation	0.51	0.688	1.35
Base of superior opening	0.342	0.427	1.25
Load application point	0.193	0.182	1.06

Table 5
Displacements when the "Sta. María" bell turns

Point	Max static (mm)	Max dynamic (mm)	Dynamic amplification factor
Coronation	0.318	0.6335	1.99
Base of superior opening	0.217	0.3878	1.79
Point application of the load	0.201	0.2898	1.44

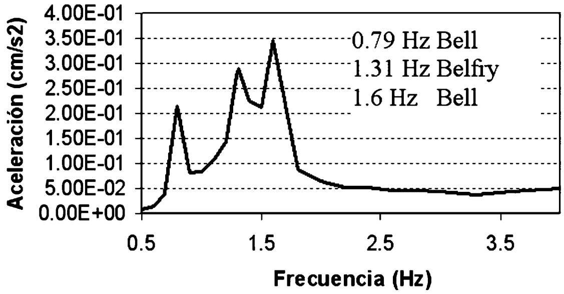

Fig. 7. Wall belfry frequency analysis of the response in the point of coronation when "Sta. María" turns.

Table 6
Displacements when the "Felipe" bell turns

Point	Max static (mm)	Max dynamic (mm)	DAF
Coronation	0.178	1.347	7.57
Base of superior opening	0.104	0.792	7.61
Point application of the load	0.139	1.065	7.66

where F = normalised horizontal force on the belfry; DAF = dynamic amplification factor; H_{max} = Horizontal force—Table 1; and P = bell weight—Table 1.

After this the case when the "Sta. María" bell is rotating was studied; the results are presented in Table 5. From this table we obtain that the total horizontal force introduced by the "Sta. María" bell for a quasi-static study is 0.56 times its weight, 709 N.

A frequency analysis of the simulated acceleration function on the coronation point of the wall belfry—Fig. 7—allows to evaluate the proximity of the structure frequency respect to the excitement frequencies. This proximity produces the increment of the dynamic amplification factor for this bell (Ivorra, 2003).

Then, in the numeric model, the case when "Felipe" bell turns was studied; the results are presented in Table 6.

The influence of the proximity of the excitement frequency respect the first mode of vibration of the structure becomes patent with this bell. The first analytical harmonic frequency is 1.195 versus the measured first harmonic frequency for this belfry of 1.359; this fact induces an increasing of the DAF. This bell induces on the wall belfry a horizontal force of 1.43 times its weight, 549 N.

4. Particular cases

After the previous cases, we study the situation in which the three bells turn together. It will be considered that the "Sta. Teresa" bell rings during 104.43 s, the "Sta. María" bell during 130.5 s and the "Felipe" bell during 154.71 s.

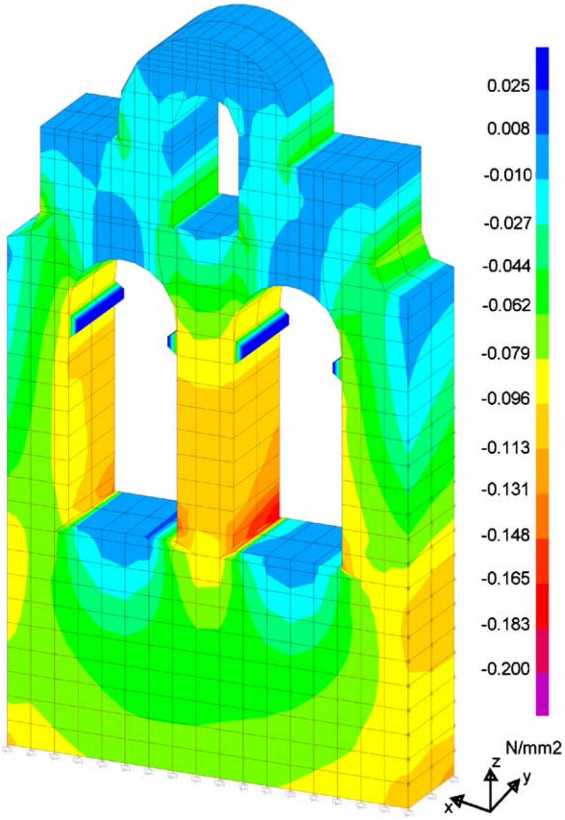

Fig. 8. Normal maximum stress distribution, σ_Z, on the wall belfry, when "Sta Teresa", "Sta. María" and "Felipe" turn together.

The distribution of normal stresses in the direction of the *z* axis when the bells turn together is presented in Fig. 8. These values are perfectly acceptable for the brick masonry.

We have considered this rhythm because is the usual one in most of the towers of our county. The smaller weight bell concludes the rotation of the group. We suppose the least favourable situation in which the three bells turn in the same direction. In this case the maximum displacement in the direction of the point of coronation of the wall belfry is 1.97 mm (Fig. 9).

We introduce the equivalent static forces in the numerical model to evaluate the corresponding quasi-static analysis. The displacement of the point of coronation of the structure is 2.47 mm. For this situation the maximum values of normal stresses in each direction are presented in Table 7.

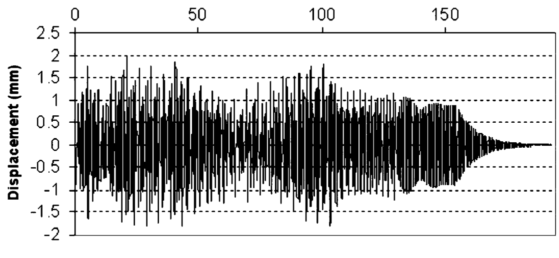

Fig. 9. Displacement of the point of coronation of the wall belfry when the three bells turn together.

Table 7
Maximum stresses in the wall belfry when the tree bells swing together

Point	σ_x (N/mm^2)	σ_y (N/mm^2)	σ_z (N/mm^2)	σ_{VM} (N/mm^2)
689	0.083	0.010	0.005	0.076
413	0.048	0.024	0.026	0.029
740	−0.026	0.016	0.040	0.058
484	−0.016	−0.016	−0.200	0.190

689 "Sta. María" bell opening, arch; *413* "Sta. María" bell opening (base), point of union with the central column; *740* Connection of "Sta. María" bell with the wall belfry in the interior part; ***484*** "Sta. María" bell opening (base), point of union with the central column.

We can admit, as a first approach, that the tension strength of the brick masonry is about 3.5 N/mm^2 and the compression strength is 120 N/mm^2. The values obtained in Table 7 are perfectly acceptable.

In a second phase the improbable situation in which the heaviest bell increases its speed frequency to values near to those of the characteristic frequency of the structure: 1.3125 Hz (8.24 rad/s) was studied. For this situation the value of the horizontal and vertical forces induced by this bell on its supports is shown in Table 8.

The displacements caused by this new variable horizontal force have been analysed from a static point of view as well as from a dynamic pont of view, the results are presented in Table 9.

The dynamic amplification factor at the point of application of the load is near 50. We will consider a horizontal force in the point of application of the load of 143.000 N for a quasi-static analysis.

In this situation the numerical model presents the collapse of the structure. In numerous points of the wall belfry the normal stresses in the vertical direction are much higher than to the acceptable 3.5 N/mm^2 of tensile strength of the material (Table 10, Fig. 10).

In this situation the point of coronation of the wall belfry has a horizontal displacement of 0.18 m. This is an unacceptable value on a belfry with only 7.54 m height.

Table 8
Sta. Teresa bell at 8.24 rad/s

Swing velocity (rad/s)	8.24
Max horizontal force (N)	2860
Max vertical force (N)	5824

Table 9
Displacements at diverse points of the wall belfry when "Sta. Teresa" bell turns at 8.24 rad/s

Point	Max static (mm)	Max dynamic (mm)	DAF
Coronation	2.24	84.48	37.71
Base of upper opening	1.46	97.8	66.99
Point application of the load	1.18	58.9	49.90

Table 10
Maximum stresses in the wall belfry when "Sta. Teresa" bell turns at 8.24 rad/s

Joint	σ_x (N/mm^2)	σ_y (N/mm^2)	σ_z (N/mm^2)	σ_{VM} (N/mm^2)
1488	7.20	21.60	12.70	13.60
466	2.86	16.70	1.91	21.60
1460	6.59	2.26	13.70	14.40
466	2.88	16.60	1.84	23.00

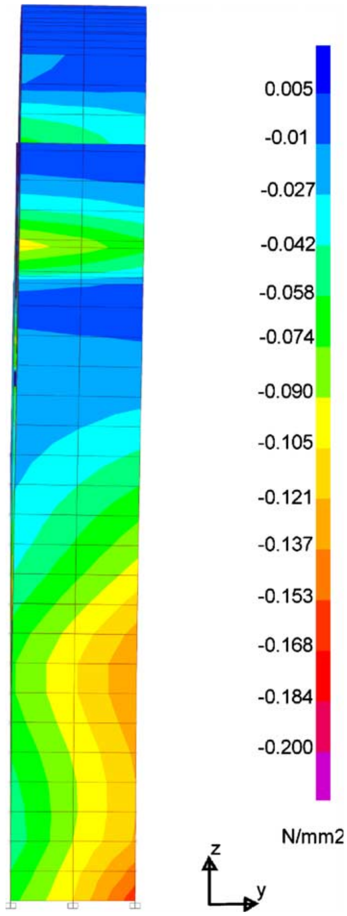

Fig. 10. Normal maximum stress distribution, σ_Z, and deformed shape when "Sta. Teresa" bell swings at 8.24 rad/s.

In a real situation, the bells ringing with the Spanish system never exceeds the turning speed of 6.28 rad/s. This is only a valid hypothetical situation for a numerical simulation. This can be a hypothetical situation when an error takes place in the programming of the automatic systems to swing bells.

5. Conclusions of the study on the wall belfry

In the belfry of Our Lady of the Carmen Church the bells turn at some coherent speeds with their own weight and their proximity to the frequency characteristic of the wall belfry. In this case, the smallest bell, "Felipe", "a priori" the one that produces less horizontal forces does not produce destructive effects, in spite of its important dynamic amplification factor: 7.54. We have even checked that the biggest bell placed in the smallest opening and rotating at its corresponding speed does not introduce excessive stresses on the system. These stresses cannot produce problems on the wall belfry.

The true negative effect would take place if the rotating speed of the bell was increased up to nearly the structure's first vibration mode. In this situation the effect of bending would increase the stresses in the structure which would originate cracks in diverse points of the wall belfry. The appearance of these cracks would imply a modification of the dynamic response of the system and therefore it would move again away from the excitement frequency introduced by the bell. Since the masonry built structure possesses a non-linear behaviour, the prediction of the response after that cracking process would need a new monitoring of the system to evaluate its new frequencies.

The shear effects or the lack of adherence of the brick masonry is compensated by the dead weight of the belfry: 387,858 N, out of which 145,492 kN are above the axes of the ‘‘Sta María’’ and ‘‘Sta. Teresa’’ bells. The total quasi-static force calculated for both bells is as low as 2080 N. Likewise on the ‘‘Felipe’’ bell, a maximum horizontal force of 549 N is shown vis-a-vis to 25,908 N of brick masonry weight above the bell axis.

As in most of the studied cases, for the Spanish system bells, the characteristic weight of the structure is the fundamental factor that allows supporting the efforts introduced by the bells and it allows its structural integrity, since the bells are very balanced.

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